

# Report on the activities of the Working Group

## Vertical Datum Standardisation

A common initiative of

GGOS Theme 1:  
Global Height System

International Gravity  
Field Service (IGFS)

IAG Commission 2:  
Gravity Field

IAG Commission 1:  
Reference Frames

Initial members

L. Sánchez (Germany), chair

R. Čunderlík (Slovakia)

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N. Dayoub (Syria)

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Y. Wang (USA)

J. Ågren (Sweden)

VERTICAL DATUM STANDARDISATION

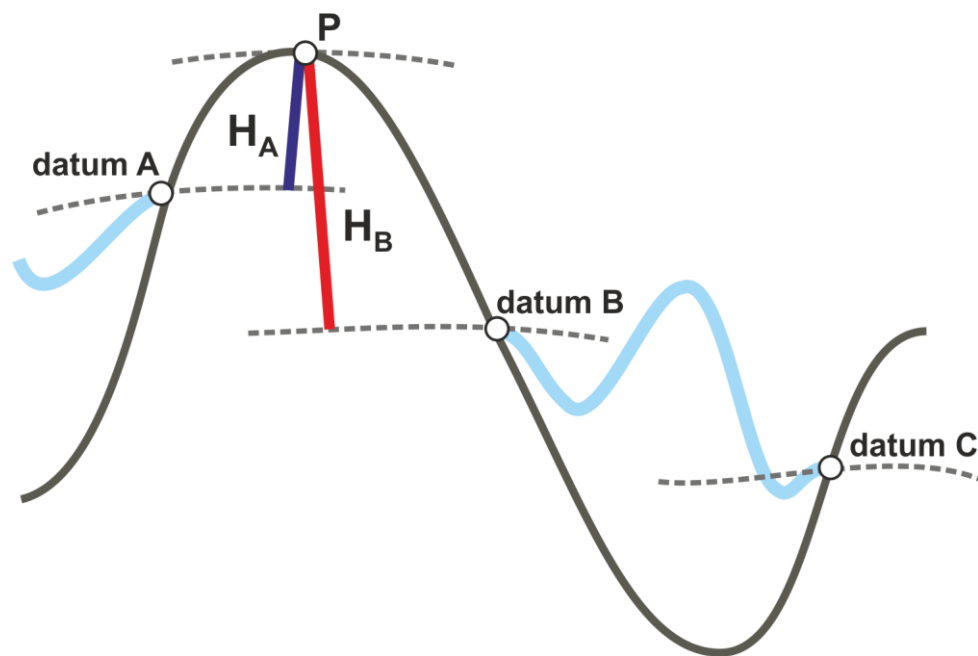
# Motivation 1: inconsistent height systems

The Global Geodetic Observing System (GGOS) requires geodetic reference frames with

- an order of **accuracy higher** than the magnitude of the phenomena and effects we want to study (e.g. global change);
- **consistency** and **reliability worldwide**;
- **long-term stability**.

## The existing height systems

- refer to **different levels** (many [dm] of discrepancy);
- realise **different types of heights** (normal, orthometric, etc.);
- omit (sea and land) **vertical variations** with time;
- do not support the precise combination of **h-H-N (= ?)**



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# Motivation 2: new methods for height determination

## Today

Levelling in combination with gravity reductions

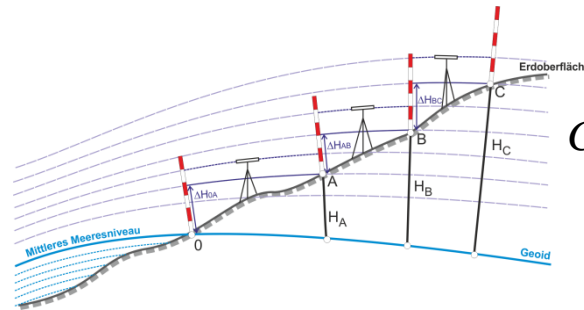
## Desired

Disturbing potential in combination with a reference ellipsoid

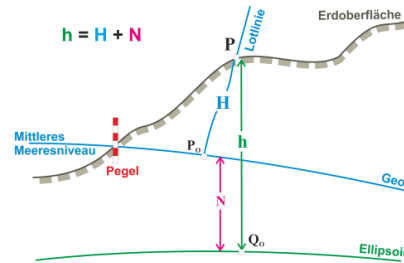
## In the future

Global gravity field models in combination with ITRS/ITRF coordinates

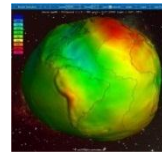
Comparison of clock frequencies of high-precision



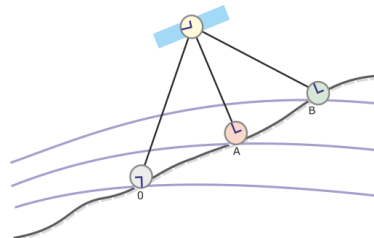
$$C(g, dn) = W_0 - W_P = \int_0^P g \, \delta n \cong \sum_0^P g \, dn$$



$$C(U_0, T) = -(U_0 - W_0) + \bar{\gamma}(\varphi)h - T(\varphi, \lambda, h)$$



$$C(\bar{C}_{nm}, \bar{S}_{nm}) = W_0 - [V(r, \theta, \lambda) + Z(r, \theta)]$$



$$C(f) = c^2 \left( \frac{f - f_0}{f_0} \right)$$

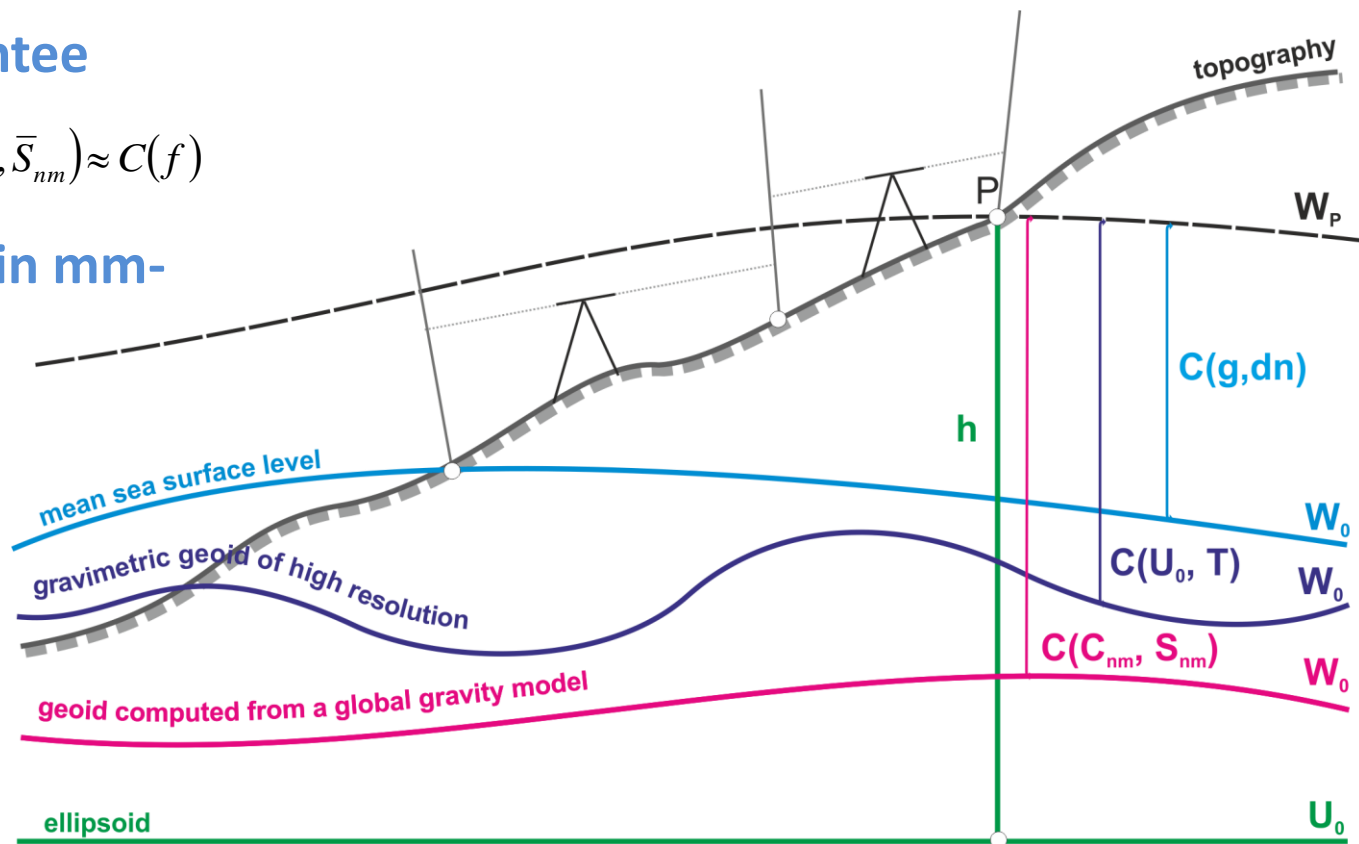
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# Reference level depending on input data?

How can we guarantee

$$C(g, dn) \approx C(U_0, T) \approx C(\bar{C}_{nm}, \bar{S}_{nm}) \approx C(f)$$

in cm-level (better in mm-level), globally ?



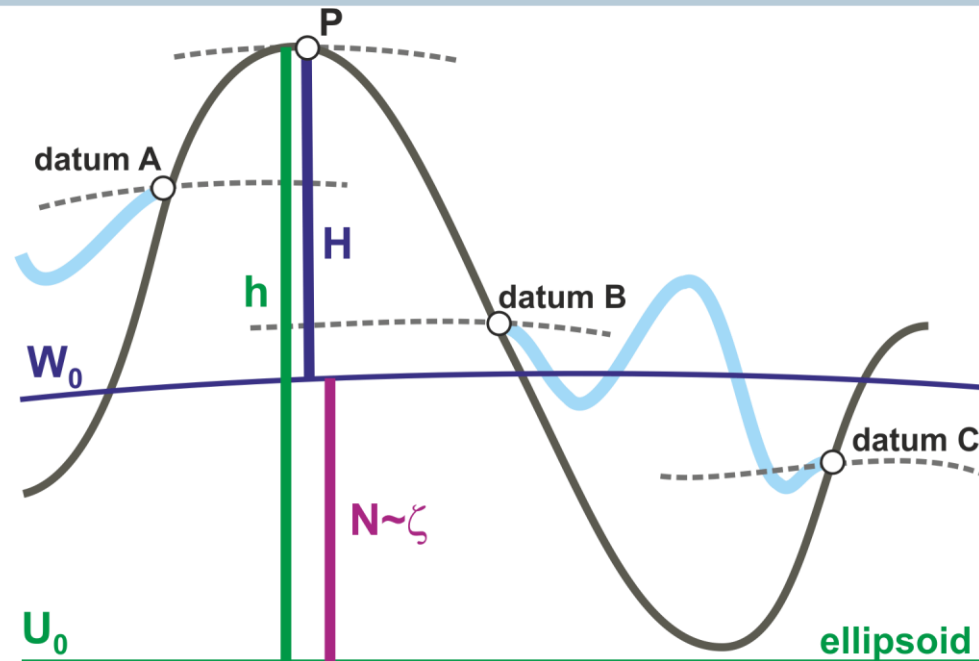
- The same  $W_0$  value for all existing (regional) geoids?
- The same geoid with different (regional)  $W_0$  values?
- Only one geoid with only one  $W_0$  value?

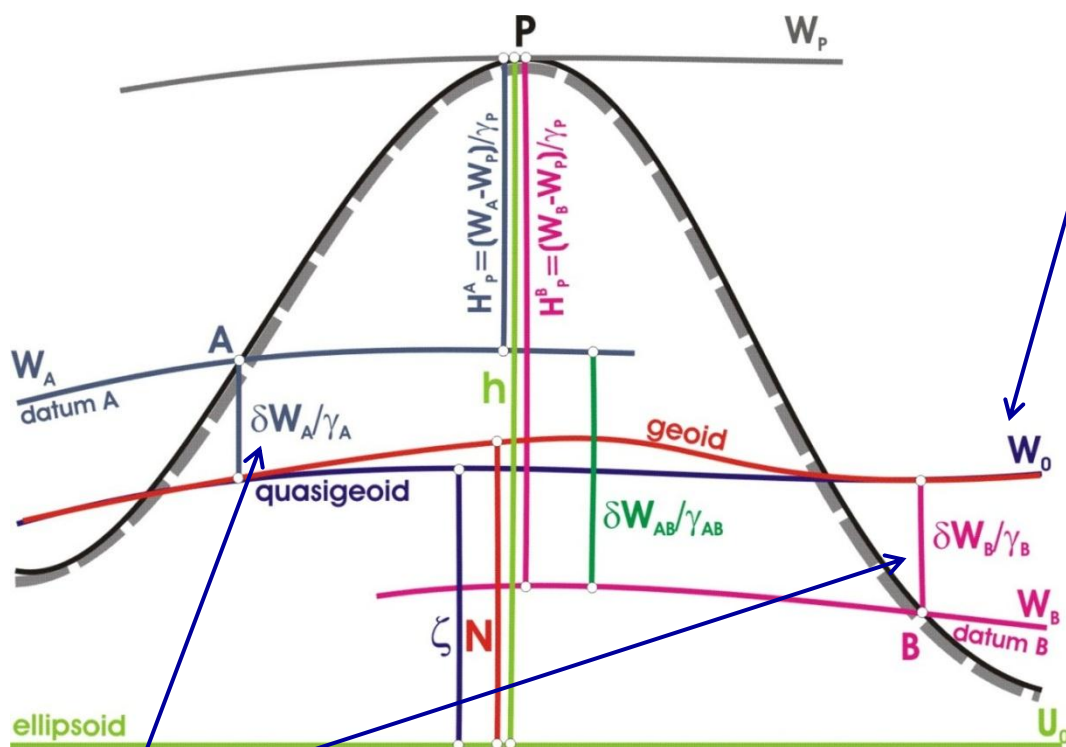
## A global vertical reference system

- To solve the **discrepancies** between the **existing height systems** and
- To support the **different techniques for height determination**.

## Implicit characteristics:

- One reference level ( $W_0$  or geoid) to be used globally;
- All existing geo-potential numbers (physical heights) referring to one and the same global level;
- Precise combination with geometric heights and geoid models of high resolution, i.e.  $h-H-N=0$ .





1. Selection (**Definition and realisation**) of a global reference level  $W_0$

- $W_0$  = potential of the geoid
- Geoid = equipotential surface best fitting the global mean sea (Gauss definition)

GGHS 2012, Section 5  
(Thursday morning)

2. Connection of the individual reference levels with the global  $W_0$

- Basic approach: 
$$h - H - N = \frac{\delta W}{\gamma}$$

GGHS 2012, Section 5  
(Wednesday afternoon)

Poster session today and tomorrow!

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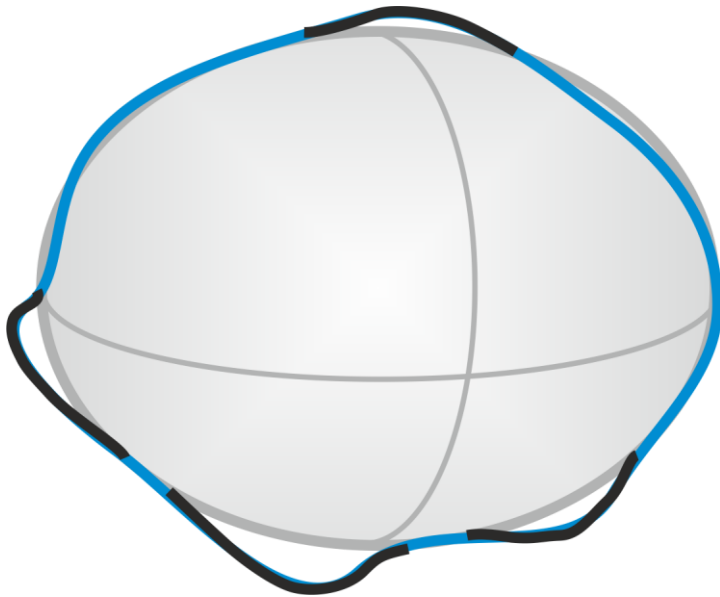
In the 1990s and before:

- Determination of the parameters for a best fitting ellipsoid

$$U_0 = U(a, f, \omega, GM); \text{ or } U_0 = U(a, J_2, \omega, GM)$$

Then by definition:

$$W_0 \stackrel{!}{=} U_0$$

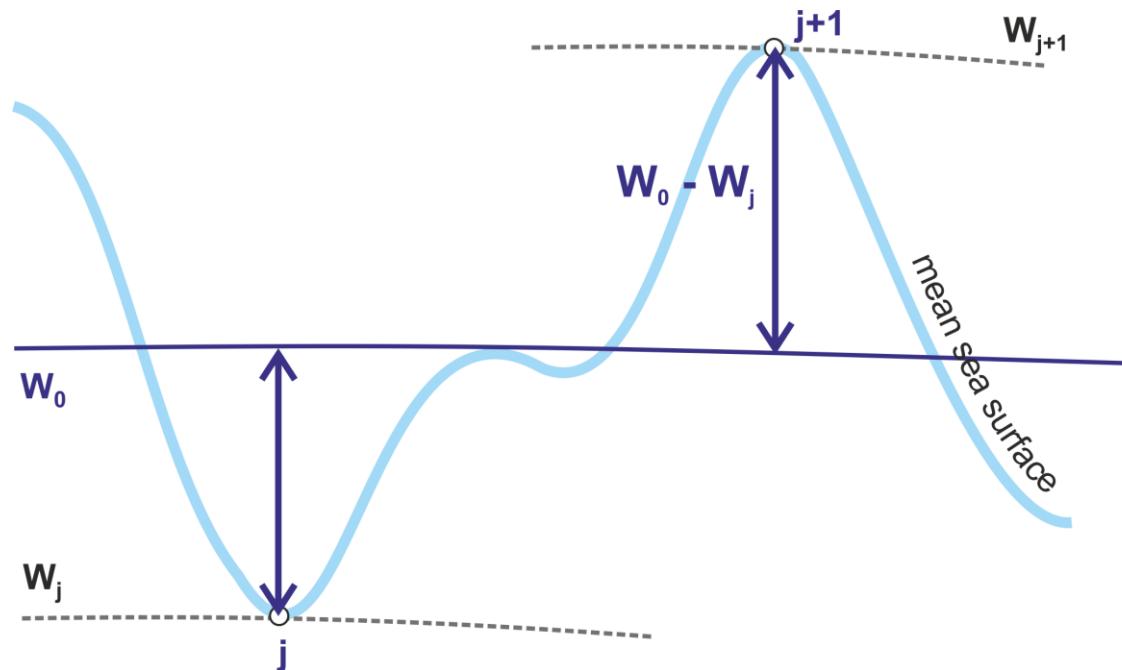


Late 1990s and 2000s:

$$\int_s \Xi^2 ds = \min; \quad \Xi = \frac{W_0 - W_j}{\gamma_j}$$

$\Xi$  : Sea surface topography

- Points  $j$  with coordinates from satellite altimetry describe the mean sea surface;
- Potential values  $W$  are derived from a global gravity model



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Today: solution of the fixed geodetic boundary value problem:

$$\nabla^2 \delta W(\mathbf{X}) = 0 \quad \mathbf{X} \in \Omega$$

$$\delta W(\mathbf{X}) \rightarrow 0 \quad \mathbf{X} \rightarrow \infty$$

$$\delta g(\mathbf{X}) = g(\mathbf{X}) - \gamma(\mathbf{X}) \quad \mathbf{X} \in \Sigma$$

Boundary surface  $\Sigma$  known;

Unknown: disturbing potential  $\delta W (=W_0 - U_0)$

Boundary condition: gravity disturbances  $\delta g$

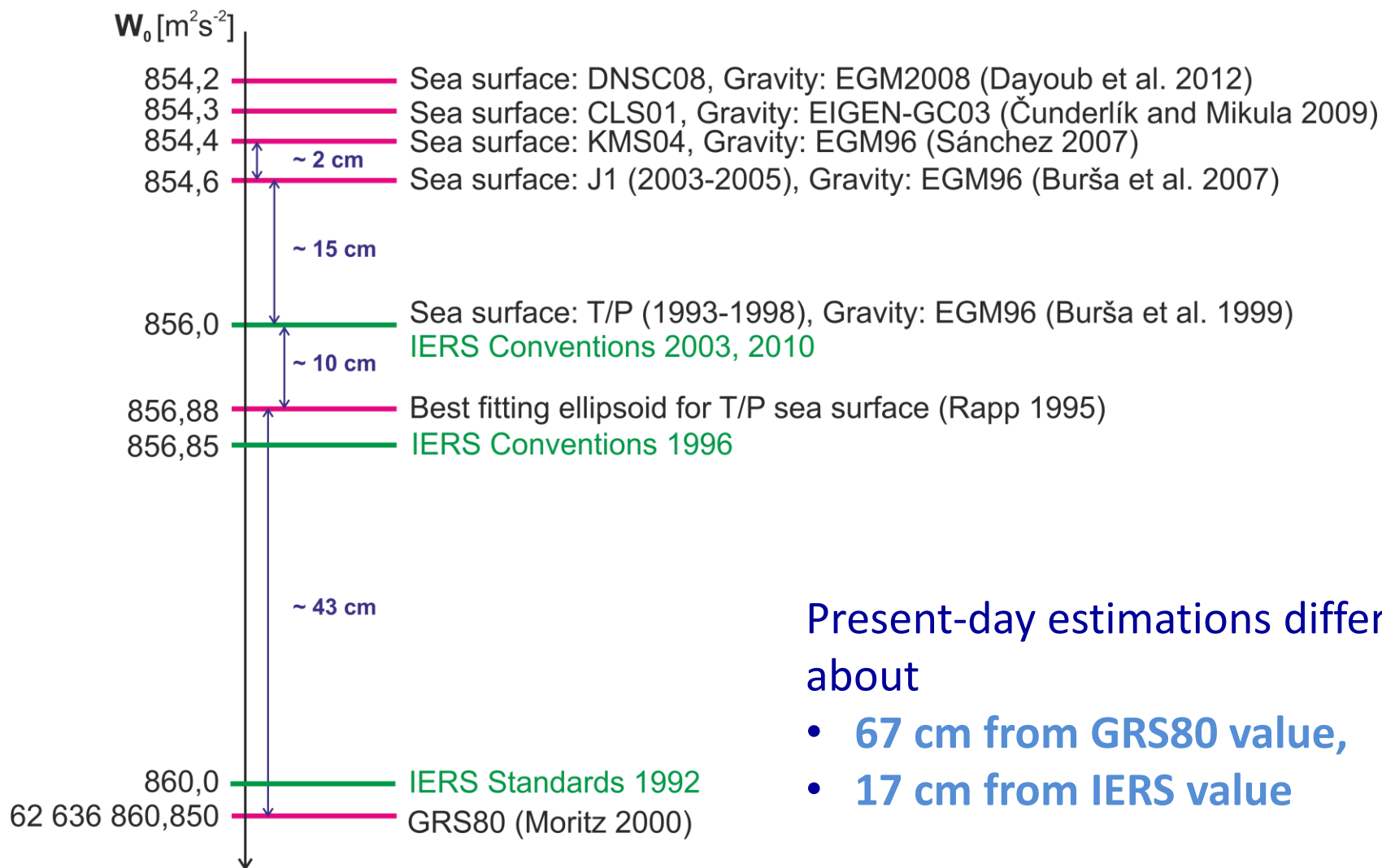
Regularisation:  $\delta W$  vanishes at infinity

$\mathbf{X} \leftrightarrow$  sea surface from satellite altimetry, continental surfaces from SMRT

$g(\mathbf{X}) \leftrightarrow$  global gravity model

$\gamma(\mathbf{X}), U_0 \leftrightarrow$  GRS80

# Some examples of $W_0$ estimates



Present-day estimations differ about

- 67 cm from GRS80 value,
- 17 cm from IERS value

- The reference level  $W_0$  for potential differences **can arbitrarily be appointed**. However, to get the worldwide consistency desired within a global vertical reference system, the selected  **$W_0$  value must be realisable** with **high-precision at any time and anywhere** around the world.
- Since  $W_0$  represents only one quantity and it is not sufficient to estimate position and geometry of the equipotential surface it is defining; **the main problem to solve here is not the determination of the  $W_0$  value *per se*, but its realisation**.
- Therefore, it is necessary to estimate it from **real observations** of the Earth's gravity field and surface.
- The **uniqueness, reliability and repeatability** of the global reference level  $W_0$  (or global geoid) can only be guaranteed by introducing **specific conventions** (like any other reference system!). On the contrary, there will exist as many height systems as  $W_0$  computations.

## Objectives

- To **bring together all teams working on the computation of  $W_0$**  to elaborate an inventory describing individual methodologies, conventions, standards, and models presently applied;
- To implement **a new  $W_0$  computation** following individual (own) methodologies, but applying the same input geodetic models;
- To make a proposal for a **formal IAG/GGOS convention about  $W_0$**  supported by a document containing the detailed computation of the recommended value.
- To provide **a standard about the usage of  $W_0$  in the vertical datum unification** describing an appropriate strategy to connect (unify, transform) any local height system with the global  $W_0$  reference level.

## On going-activities

L. Sánchez (Germany)

⇒  $W_0$ -computation based on fixed-GBVP, analytical solution

R. Čunderlík (Slovakia)

Z. Faskova (Slovakia)

K. Mikula (Slovakia)

⇒  $W_0$ -computation based on fixed-GBVP, Boundary Element Method (BEM), Finite Element Method (FEM) and Finite Volume Method (FVM).

N. Dayoub (Syria)

P. Moore (United Kingdom)

⇒  $W_0$ -computation based on averaging  $W$ -values from a GGM on points describing the sea surface (MSS)  
 $W_0$ -computation based on a reference ellipsoid ( $W_0 = U_0$ )

Z. Šima (Czech Republic)

V. Vatrť (Czech Republic)

M. Vojtiskova (Czech Republic)

⇒  $W_0$ -computation based on averaging  $W$ -values from a GGM on points describing the sea surface (MSS)

J. Huang (Canada)

D. Roman (USA)

Y. Wang (USA)

J. Ågren (Sweden)

⇒ Regional realisation of a global  $W_0$

## First results

The different teams computed  $W_0$  using the same input data, but their own methodologies.

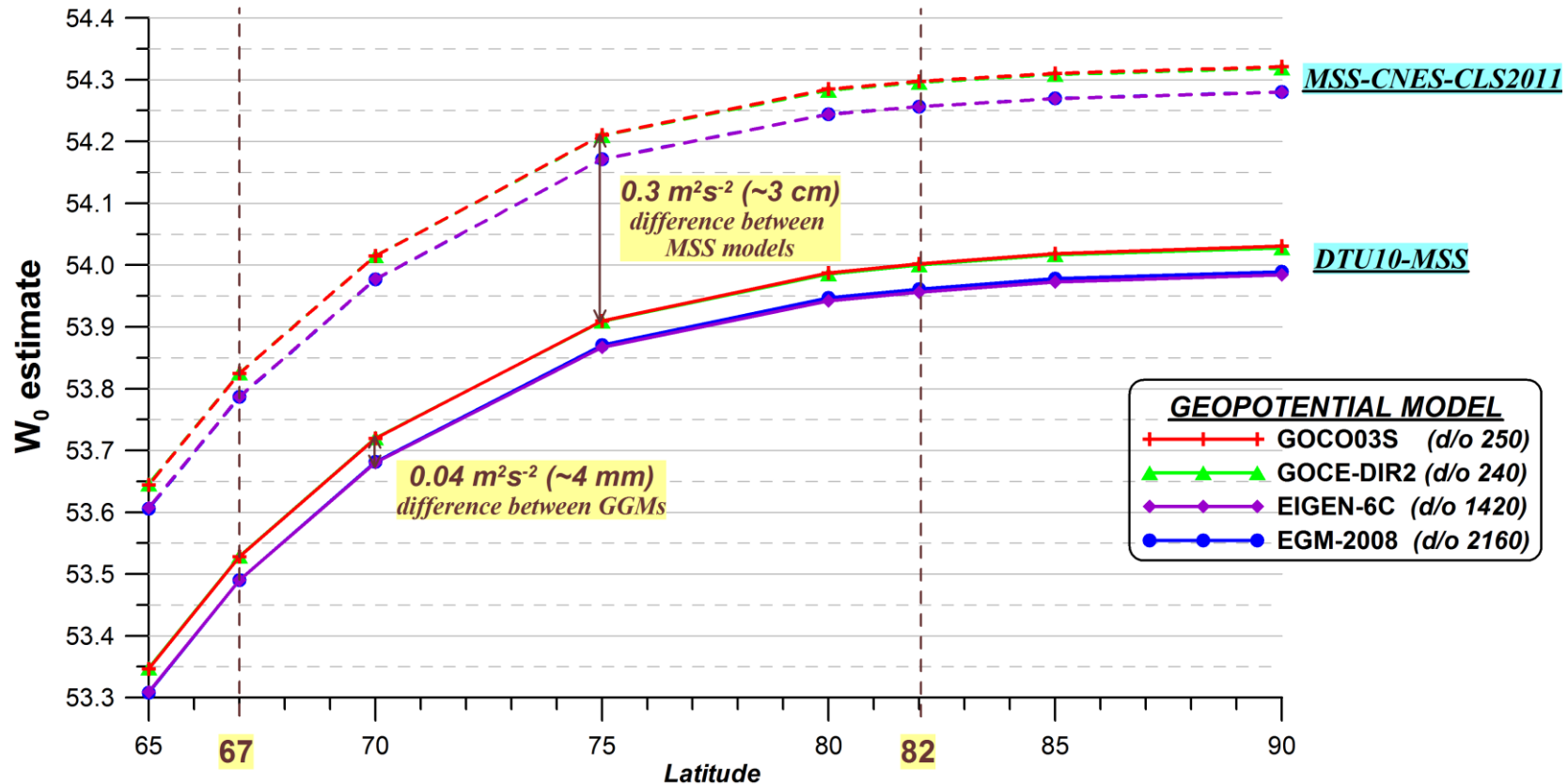
Estimates provided by N. Dayoub

MSS	Domain N/S	GGM	Max degree	$W_0(\text{m}^2\text{s}^{-2})$		
				1996.0	2001	2005
CLS11	82°/82°	EIGEN6C	$n=200$	62636854.43		62636854.19
		GOCO03S		62636854.43		62636854.19
	67°/67°	EIGEN6C		62636854.06		62636853.82
		GOCO03S		62636854.06		62636853.82
DTU10	82°/82°	EIGEN6C			62636854.11	62636854.00
		GOCO03S			62636854.11	62636854.00
	67°/67°	EIGEN6C			62636853.75	62636853.64
		GOCO03S			62636853.75	62636853.64

- $W_0$ -dependence on the latitude coverage.
- $W_0$ -dependence on the reference epoch of the mean sea surface model and potential coefficients.

## First results

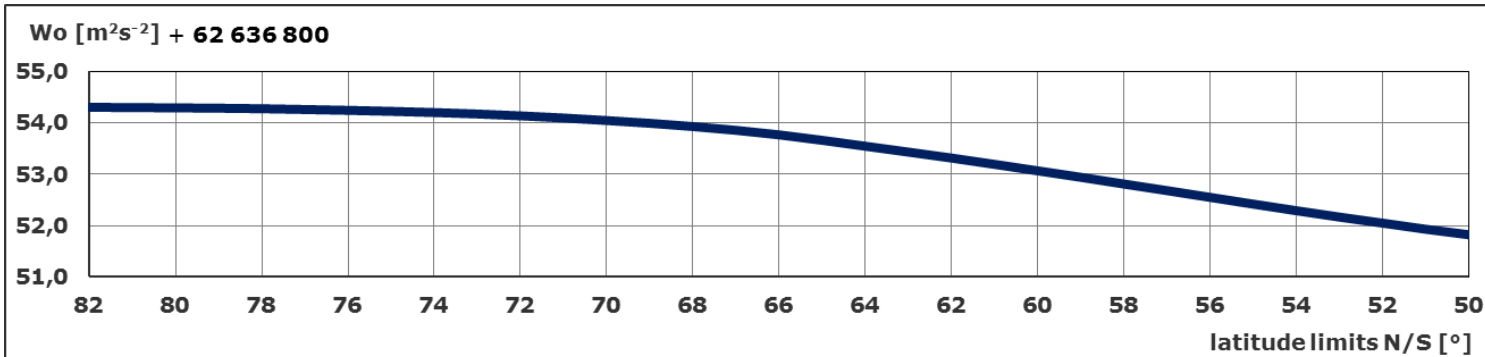
Estimates provided by R. Čunderlík, Z. Faskova, K. Mikula



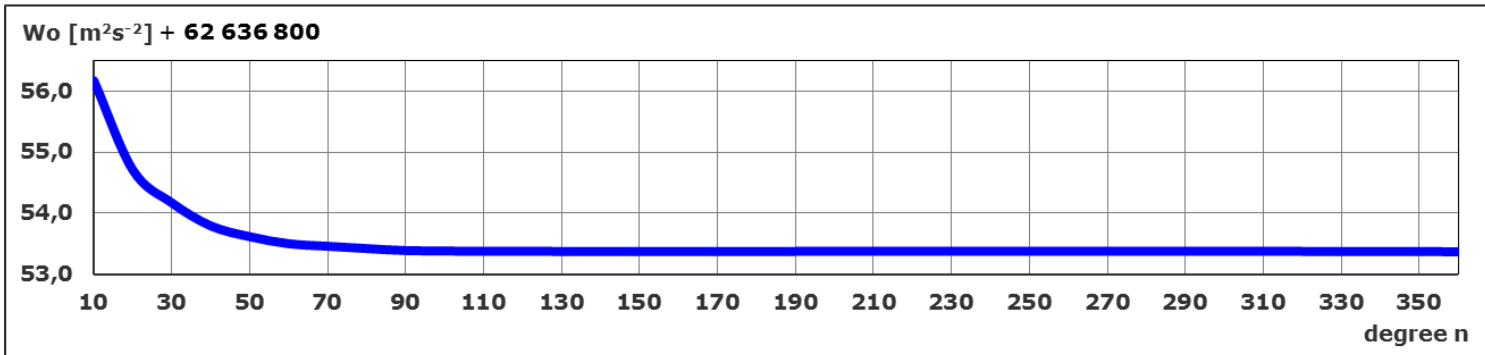
- $W_0$ -dependence on the spectral resolution of the gravity model.

# WG on Vertical Datum Standardization

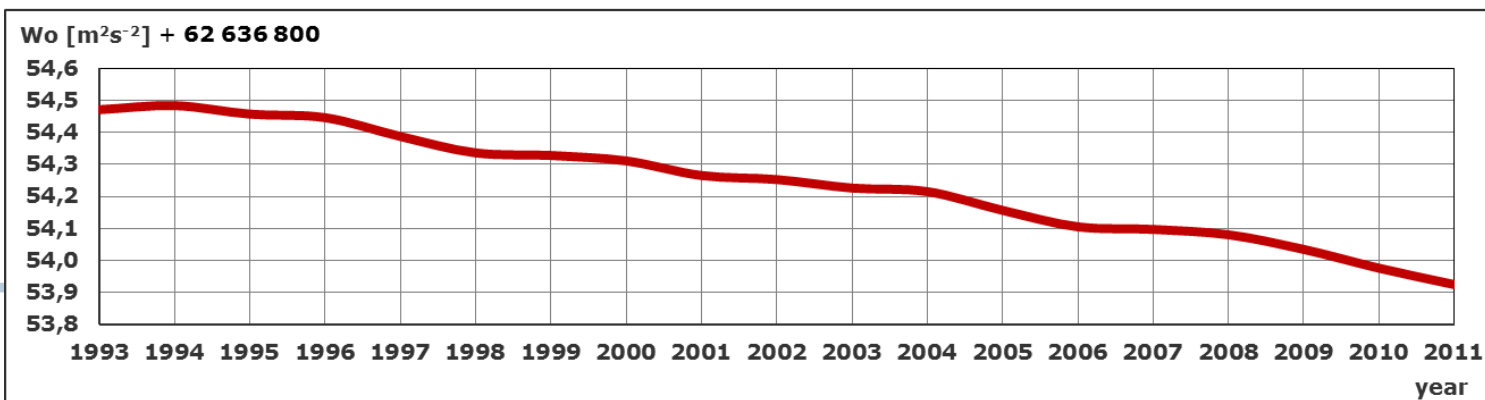
## First results Estimates provided by L. Sánchez



$W_0$ -variation with latitudinal coverage.



$W_0$ -variation with degree  $n$  of the GGM.



$W_0$ -variation with time.

LDATUMSTANDARDISATION

- All the computations are delivering very close results, but there are still differences of about  $0,5 \text{ m}^2\text{s}^{-2}$  ( $\sim 5 \text{ cm}$ ). It is necessary to start defining the standards and conventions for a formal recommendation on  $W_0$ .
- Activities to be faced in the close future:
  - Combination of a “geodetic” sea surface model and an “oceanographic” DOT-model to reproduce a sea surface closer to an equipotential surface (geoid);
  - Integration of polar regions on the Earth’s surface representation;
  - Differences between  $W_0$  values obtained from a long-term mean sea surface model and yearly mean sea surface models;
  - A formal procedure for the error propagation analysis.

**Splinter Meeting @ GGHS 2012: Thursday, Oct. 11, 6:15 pm. Room 8.**

To join the group visit <http://whs.dgfi.badw.de> or send a message to [sanchez@dgfi.badw.de](mailto:sanchez@dgfi.badw.de).