The permanent tide and the International Height Reference System IHRS

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Contents

- review of issues how to handle the mean tide in IHRS/IHRF, proposals
- review of formulas new and old
- key proposal: compute everything in zero-tide system, transfer to mean-tide at the very end, using simplified formulas
- this will keep the computations consistent with the gravity/geoid work in zero-tide without introducing an awful amount of new transformations and corrections

IAG Resolution (No. 1) for the definition and realization of an International Height Reference System (IHRS)
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- 2. parameters, observations, and data shall be related to the mean tidal system/mean crust

Parameters, observations, and data shall be ***related*** to the mean tidal system/mean crust

- on many occasions I have maintained that this is no major disruption as we have or have had national and continental height systems in the mean tidal system
- and subsequently have experience and formulas for handling them
- Is this strictly true?
- Well, it depends
- If we take the phrase very rigorously, it could become a major disruption / an unholy mess
- The key word is ***related*** which allows sensible strategies

My major points/proposals I

1. Unify formulas starting from [the IERS value for] the time-average W_2 for the tide-generating potential
2. Ignore the height-dependency of W_2
3. Related to (2): Do NOT introduce mean-tidal gravity for computing geopotential differences from levelling results, NOR for converting geopotential numbers to orthometric heights

My major points/proposals II

4. Computation of normal heights from geopotential numbers in the mean tidal system is theoretically consistent only when the normal gravity used is generated by a reference field that includes the mean tidal potential (in addition to gravitational and centrifugal potential).
5. Some of the (a, f, J_2, U_0) would come out different from the corresponding values in the standard Pizzetti theory
6. If done, this would create the Unholy Mess
7. DO NOT DO IT.
8. Instead, define mean-tidal normal heights as a simple ”datum transformation” to zero-tide normal heights

My major points/proposals III

9. The proposals (1-3, 7) imply that the mean-tide IHRS for geopotential numbers (at least in its realizations IHRF) would be simply a zero-tide IHRS/IHRF with the latitude-dependent datum transformation by adding the $-W_2$ value corresponding to the surface of the ellipsoid
10. I believe the datum is the whole point of the exercize, other issues are just unwanted byproducts?
11. Think about standardizing
    $W_2/\bar{g}$ and $W_2/\bar{\gamma}$
    values in order to have simple (= depend on latitude but not height) additive terms between zero and mean-tide heights for orthometric and normal heights as well

Closer examination of some of the items

For second-degree tides, the time-average of the summed tide-generating potential of celestial bodies can be written in the form

$$W_2(r,\psi) = B\left(\frac{r}{R}\right)^2 P_2(\sin\psi)$$

where

- (r, ψ) are the geocentric radius and latitude
- $P_2(.)$ is the second-degree Legendre polynomial
- R is scaling factor for distances
- B is a coefficient (that depends on R)

1. Standardising the formulas

Consistent formulas for all quantities related to the permanent tide can (***should***) be obtained from a (conventional, best) formula for the time average $W_2(r,\psi)$ of the tide-generating potential

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Different normalizations for the coefficients exist. IERS Conventions (2010) use the formulation

$$W_2(r,\psi) = B\left(\frac{r}{R}\right)^2 P_2(\sin\psi) = H_0\sqrt{\frac{5}{4\pi}}g_e\left(\frac{r}{R_e}\right)^2\left(\frac{3}{2}\sin^2\psi - \frac{1}{2}\right)$$

with

H_0 = –0.31460 m

R_e = 6378136.55 m

g_e = 9.79828685 ms^{-2}

For easier comparison, write all in the form

$$W_2(r,\psi) = A\left(\frac{r}{a}\right)^2\left(\sin^2\psi - \frac{1}{3}\right)$$

where a is the semimajor axis of the GRS80 ellipsoid

and then compare the coefficients A

Question: Where do we get accurate coefficient for $W_2(r,\psi)$?

Answer: From the time-independent terms ($M_0 S_0$) of a time-harmonic expansion of the tide-generating potential

Coefficient A of the term $\sin^2\psi$ when $R = a$ = semi-minor axis of GRS80 ellipsoid, epoch 2000.0
Rate in the coefficient is –0.0009 m^2s^{-2}/century

Authors	Year	Method	
Cartwright-Tayler-Edden	1973	Numerical	-2.91652 m^2s^{-2}
Büllesfeld	1985	Numerical	-2.9164(0)
Tamura	1987	Numerical	-2.91656
Xi	1987	Analytical	-2.91647
Hartmann-Wenzel	1995	Numerical	-2.91656
Roosbeek	1995	Analytical	-2.91665
Kurdryatsev	2004	Numerical	-2.91658
IERS Conventions	2003	One of the above	-2.9166(2)
Adopted for EVRS	2008	IERS Conventions	-2.9166

Numerical = spectral analysis of time series generated using numerical ephemeris
Analytical = algebraic manipulations
Adopt the IERS conventions

Some formulas deduced using the IERS coefficient

W_2 in geodetic coordinates (φ, h) close to the surface of the GRS80 ellipsoid, unit m^2s^{-2}

$$W_2(\varphi,h) = \left(1 + 0.31\times 10^{-6}\,\mathrm{m}^{-1}\,h\right)\left(0.9722 - 2.8841\sin^2\varphi - 0.0195\sin^4\varphi\right)$$

Gravity contribution of W_2 in geodetic coordinates, unit μGal

$$g_2(\varphi) = -30.49 + 90.95\sin^2\varphi + 0.31\sin^4\varphi$$

Dependence on h is very small and is not shown

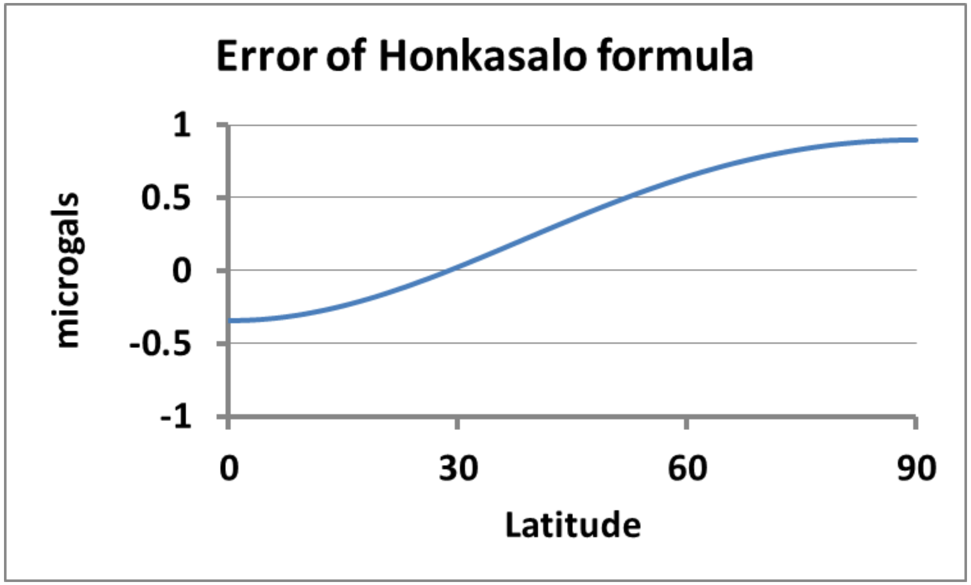
A detour: some widely used legacy formulas for the permanent tide and related quantities give rather accurate results

Widely used:

- Honkasalo (1964): calculation of the mean value in time of the tidal correction to gravity used at the time
- Zadro-Marussi (1973): calculation for $W_2(r,\psi)$ quoted by Burša et al in the 1990s and by Groten (2000, 2004)
- Heikkinen (1978): calculation of the gravity contribution g_2 of the W_2, of the mean horizontal tide-generating force, and of the W_2 itself at the surface of the GRS67 ellipsoid
- Ekman (1989): a range of formulas based on Heikkinen (1978)

Example 1: Honkasalo (1964) for mean tide gravity contribution (originally written as mean correction using gravimetric factor 1.20), unit µGal

$$g_2(\varphi) = 111\left(\sin^2\varphi - \frac{1}{3}\right)\Big/1.20$$



Example 2: Zadro and Marussi (1973)

$$W_2(r,\psi) = -1.94\left(\frac{r}{R}\right)^2\left(\frac{3}{2}\sin^2\psi - \frac{1}{2}\right)$$

in m^2s^{-2}, with $R = 6371$ km the mean radius of the Earth. With $R = a$ the coefficient of $\sin^2\psi$ becomes -2.9165 or very close to the IERS value -2.9166

This however seems like a lucky roundoff; I have calculated more digits using the formulas by Zadro-Marussi and updated astronomical constants. The coefficient is then -1.9391 and $\sin^2\psi$ at $R = a$ gets -2.9151

In any case, the difference to IERS corresponds to less than 0.2 mm.

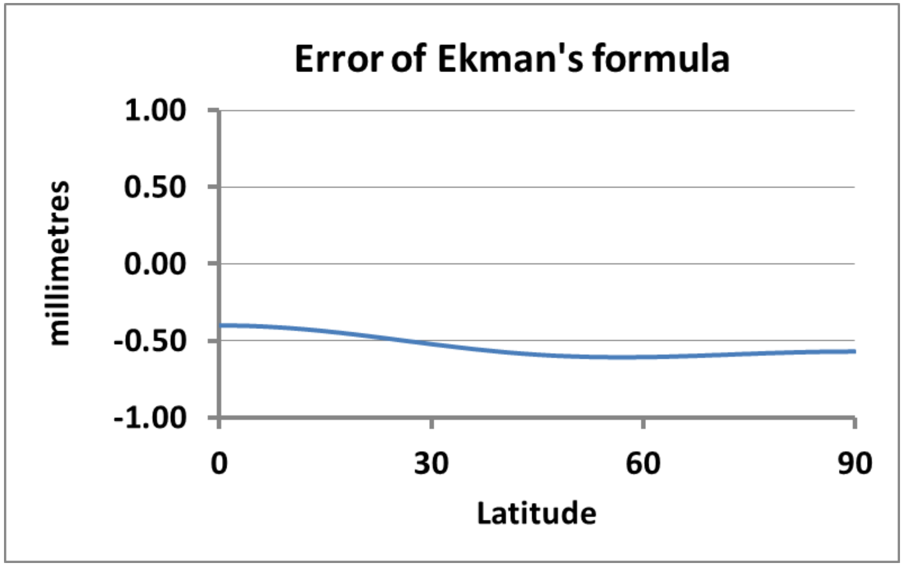
Example 3: Ekman (1989), effect of roundoff

Ekman (1989) wrote using the results by Heikkinen (1978), for the difference between the mean-tide geoid and the zero-tide geoid, in mm

$$N = \frac{W_2}{g} = 99 - 296\sin^2\varphi$$

Compare this with the formula in EVRS standards (2008)

$$H_2(\varphi) = W_2(\varphi)/\gamma_0(\varphi) = +99.40 - 295.41\sin^2\varphi - 0.42\sin^4\varphi$$



End of detour

Mean-tide geoid and zero-tide geoid

Let $W(X)$ be the zero tide potential and $W_m(X)$ the mean-tide potential . Further, let $C(X)$ be the zero-tide geopotential number and $C_m(X)$ the mean-tide geopotential number. X means here any 3-D coordinate triple (could be 3-D Cartesian, 3-D ellipsoidal, etc.)

We have $W_m(X) = W(X) + W_2(X)$

The zero geoid is defined by the surface

$$W(X) = W_0$$

and the mean geoid by the surface

$$W_m(X) = W_0 \ , \qquad \text{i.e.,} \quad W(X) + W_2(X) = W_0$$

Mean-tide heights and zero-tide heights

Let $C(X)$ be the zero-tide geopotential number and $C_m(X)$ the mean-tide geopotential number.

Zero tide geopotential numbers are defined by

$$C(X) = -[W(X) - W_0]$$

and mean tide geopotential numbers by

$$C_m(X) = -[W_m(X) - W_0] = -[W(X) + W_2(X) - W_0] = C(X) - W_2(X)$$

This sure looks like a simple datum transformation of zero-tide geopotential numbers!

$$C_m(X) = C(X) - W_2(X)$$

But there is a caveat:

There is potential trouble ahead (pun intended) because

The $W_2(X)$ depends on height, and

What about conversion to metric heights?

2. The height-dependence for potential:

$$W_2(r,\psi) = B\left(\frac{r}{R}\right)^2 P_2(\sin\psi)$$

and thus depends on r.

In geodetic coordinates (φ, h) close to the surface of the GRS80 ellipsoid, unit m^2s^{-2}

$$W_2(\varphi,h) = \left(1 + 0.31\times10^{-6}\,\mathrm{m}^{-1}\,h\right)\left(0.9722 - 2.8841\sin^2\varphi - 0.0195\sin^4\varphi\right)$$

calculated for EVRS conventions (2008). For $h = 10\ 000\ \mathrm{m}$ ”the scale factor” is 3×10^{-3} and its effect max $0.6\ \mathrm{mm}$

Proposal: drop the h term for IHRS/IHRF conventions

3. Mean tidal gravity?

Obviously, in a rigorous mean-tidal height system , the gradient of the *total* potential (gravitational+centrifugal+*permanent tide*) should be used to transform the levelling observations to geopotential differences.

I.e., in calculating the difference ΔC in geopotential numbers on a levelling interval , we should multiply the levelled height difference Δh_{obs} ,
not with zero-tide gravity g
but with $g+g_2$
where g_2 is the component of grad W_2 on the local vertical

Let us not do it!!

Note that this is independent of whether the tidal correction to Δh_{obs} refers to zero-tide or mean tide.

Naturally, I also advocate using zero-tide here.

Gravity contribution of W_2 in geodetic coordinates, unit μGal

$$g_2(\varphi) = -30.49 + 90.95\sin^2\varphi + 0.31\sin^4\varphi$$

Dependence on h is very small and is not shown

The mean-tide gravity differs from zero-tide gravity by about 1×10^{-7} only
Using it instead of zero-tide gravity in processing levelling data merely reproduces the (small) dependence of W_2 on height that ”we” just ”decided” is better to neglect.

3 (continued). The gravity divisor in converting geopotential numbers to orthometric heights?

Zero-tide H: the zero-tide $C(X)$ gets divided by zero-tide gravity $\bar{g}$ averaged over the plumb line

$$H = \frac{C}{\bar{g}}$$

Mean-tide H_m *:* The mean-tide $C_m(X)$=$C(X)$+$W_2(X)$ should get divided by

$$\bar{g}_m = \bar{g}' + g_2$$

averaged mean-tide gravity which in fact is zero-tide gravity averaged over a slightly different range, plus g_2, the mean-tide contribution to gravity.

$$H_m = \frac{C_m}{\bar{g}_m} = \frac{C + W_2}{\bar{g}' + g_2}$$

3 (continued). The gravity divisor in converting geopotential numbers to orthometric heights?

$$H = \frac{C}{\bar{g}} \qquad H_m = \frac{C_m}{\bar{g}_m} = \frac{C + W_2}{\bar{g}' + g_2} \approx \frac{C + W_2}{\bar{g}} = H + \frac{W_2}{\bar{g}}$$

$\bar{g}_m = \bar{g}' + g_2$ differs from $\bar{g}$ by max 1×10^{-7} only; neglect the difference.

But the dependence of $\bar{g}$ from the height creates a minor problem if we want to give a conversion formula between H and Hm that depends on latitude only.

4 The normal gravity in converting geopotential numbers to normal heights?

The mean gamma for converting the geopotential value to normal height is not just another easy-to-get gravity value

It has a basis in potential theory: a distance in the reference potential field which provides the same potential difference for the reference field as the geopotential number provides in the real field.

4 The normal gravity II

In a rigorous theory using mean-tide potential, such a normal field should be defined also as a mean-tide potential field.

Thus it should in addition to the gravitational potential and the centrifugal potential contain also the mean tide. The ellipsoidal surface would be the equipotential surface of the sum of these three potentials and subtraction the normal potential from the actual potential would leave a purely gravitational potential.

An enlarged Pizzetti theory.

4 The normal gravity III

It has been done and it works (Vermeer-Poutanen, GraGeoMar in Tokyo 1996 Proceedings).

But: some of the (a, f, J_2, U_0) would come out different from the zero-tide theory.

So, let us not do it!

Would work in connection with a completely new buildup of all reference systems, e.g. also including a realistic model atmosphere instead of condensation on the ellipsoid.

The ”mean-tide ellipsoid”

Remark: there is demand and use in the oceanographic and altimetric community for a ”mean tide ellipsoid”. However, it is usually an additionally ”flattened” Pizzetti ellipsoid, where the flattening has been increased using the J2 of the permanent tide on the surface of the Earth as if the permanent tide there would be generated by the ellipsoidal masses.

Thus there is no consistent theory behind it and it certainly cannot provide any gravity. Better consider it just an ad-hoc geometric construction.

W_2 divided by normal gravity at ellipsoid, in mm

$$H_2(\varphi) = W_2(\varphi)/\gamma_0(\varphi) = +99.40 - 295.41\sin^2\varphi - 0.42\sin^4\varphi$$

Could be the conventional system difference between the ”mean-tide normal heights” and zero-tide normal heights.

Summary

1. Calculate everything in zero tide
2. At the very end, transform to mean-tide heights, using simple datum shift
3. Ignore the fine ”print”.

Thank you!