Vertical Datum Unification for the International Height Reference System (IHRS)

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Outline

- **The GBVP approach to height datum unification**
  - Multi-datum GBVP solution
  - Observation equations and least-squares estimation
- **Factors affecting the estimation of datum offsets – NA simulation**
  - Geoid omission error
  - Indirect bias term
- **Determination of datum offsets from global W_o – SA example**
  - Available observables
  - Estimation of datum parameters
- **Conclusions**

International Height Reference System (IHRS)

Definition

- Earth-fixed geopotential reference system with Coordinates:
  - geopotential values $W(\mathbf{x})$ (and $dW(\mathbf{x})/dt$)
  - geocentric Cartesian coordinates $\mathbf{x}$ (and $d\mathbf{x}/dt$) in the ITRS/ITRF
- Parameters, observations, data in mean-tide / mean crust system

Realization

- International Height Reference Frame (IHRF) of stations with
  - $\mathbf{x}$, $d\mathbf{x}/dt$
  - $W(\mathbf{x})$, $dW(\mathbf{x})/dt$

    or, preferably,

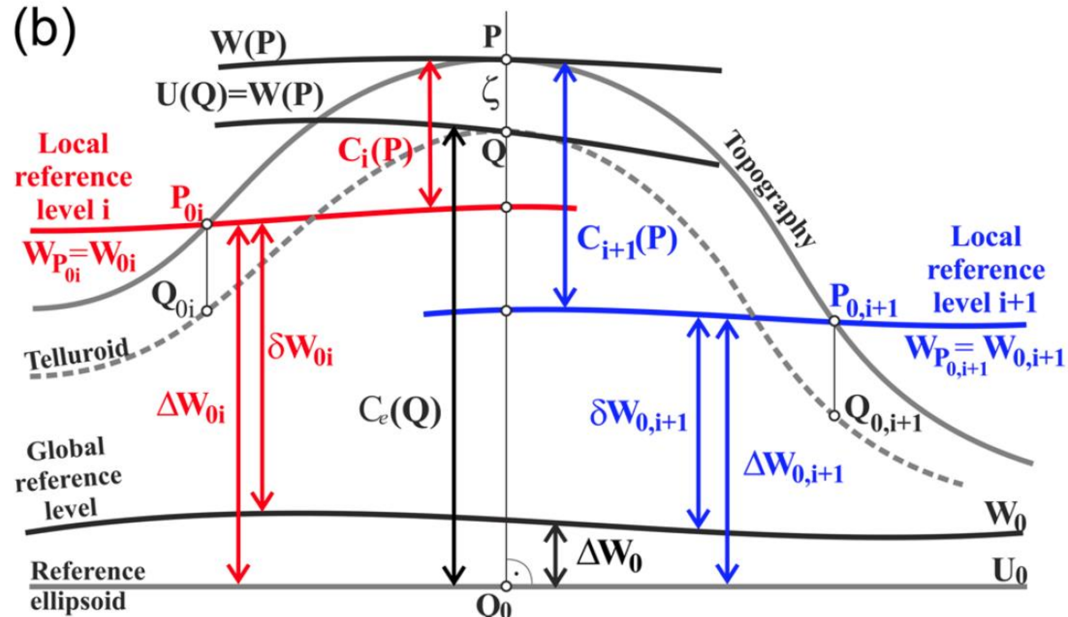
    W_o = const. = 62 636 853.4 m^{2}s^{-2} (conventional)

    $C(\mathbf{x}) = -\Delta W(\mathbf{x}) = W_o - W(\mathbf{x})$, $dC(\mathbf{x})/dt$

    - $W_P = W(\mathbf{x}_P)$ from: levelling/altimetry + gravity; GBVP solution; hi-res GGM
- Standards, conventions, procedures
  - consistency between the definition (IHRS) and the realization (IHRF)

Vertical Datum Unification Principle

◆ Illustration of the principle



source: Sanchez and Sideris, 2017

❖ **Offsets between *global datum* and each *local datum i*** (subscript *i*)

$$\delta W_{oi} / \gamma = h_P - H^*_{Pi} - \zeta_{Pi}$$ local datum height offset

GNSS or altimetry

Levelling or ocean model

GBVP solution

$$\delta W_{oi} = W_o - W_{oi} = \gamma h_P - C_{Pi} - T_{Pi}$$ local datum potential offset

The GBVP Approach (1/2)

GBVP Approach to Datum Unification

- **The global datum:** $P_o, \quad W(P_o) = W_o, \quad \Delta W_o = W_o - U_o$
  - Problem input and output (assume $M = M_e$):

$$\Delta g = -\frac{\partial T}{\partial r} - \frac{2}{R}T + \frac{2}{R}\Delta W_o \qquad T_P = -\Delta W_o + T_P^{grav} = -\Delta W_o + T_P^{GGM} + S_P \Delta g^{res}$$

Omission error

- **The local datum:** $P_{oi}, \quad W(P_{oi}) = W_{oi}, \quad \delta W_{oi} = W_o - W_{oi}$
  - Problem input and output:

$$\Delta g_i = \Delta g - \frac{2}{R}\delta W_{oi}$$

$$T_{Pi} = -\Delta W_o + \delta W_{oi} + T_P^{GGM} + S_P(\Delta g_i^{res} + \frac{2}{R}\delta W_{oi}) = -\Delta W_o + \delta W_{oi} + T_{Pi}^{grav} + \delta T_{Pi}$$

  - The datum potential offset, δW_{oi}, causes both a ***direct effect*** on ζ, $\delta W_{oi}/\gamma$, and an ***indirect effect*** $\delta T_{Pi}/\gamma$ through the biased gravity anomalies

The GBVP Approach (2/2)

◆ Estimation of *T* to be used for datum offset determination

❖ The gravimetric T in datum i using (biased) gravity data in that datum is

$$T_{Pi} = -\Delta W_o + \delta W_{oi} + T_P^{GGM} + T_{Pi}^{res} + \delta T_{Pi} = -\Delta W_o + \delta W_{oi} + T_P^{GGM} + T_{Pi}^{res} + T_{Pi}^{ind}$$

From satellite gravity models

Omission error from local gravity

Indirect Effect from all datum offsets

❖ The ***indirect effect*** T_{Pi}^{ind} is the most troublesome to compute as, for J different datums, it requires knowledge of all offsets:

$$T_{Pi}^{ind} = \mathbf{S}_P^{\Delta\sigma_i}\left(\frac{2}{R}\right)\delta W_{oi} + \sum_{\substack{j=1 \\ j\neq i}}^{J} \mathbf{S}_P^{\Delta\sigma_j}\left(\frac{2}{R}\right)\delta W_{oj}$$

$$= \delta W_{oi}\mathbf{S}_P^{\Delta\sigma_i}\left(\frac{2}{R}\right) + \sum_{\substack{j=1 \\ j\neq i}}^{J} \delta W_{oj}\mathbf{S}_P^{\Delta\sigma_j}\left(\frac{2}{R}\right) = \delta W_{oi} f_{Pi} + \sum_{\substack{j=1 \\ j\neq i}}^{J} \delta W_{oj} f_{Pj}$$

◆ Observation equations

$$\delta W_{oi} + T_{Pi}^{ind} + v_P = \gamma h_P - C_{Pi} + \Delta W_o - T_P^{GGM} - T_{Pi}^{res}$$

$$\delta W_{o,i} - \delta W_{o,i+1} + T_{Pi}^{ind} - T_{P,i+1}^{ind} + v_P = -C_{Pi} + C_{P,i+1} - T_{Pi}^{res} + T_{Pi,i+1}^{res}$$

Least-squares Estimation of Offsets

◆ Functional model

$$l_P = gh_P - C_{Pi} + DW_o - T_P^{GGM} - T_{Pi}^{res} = (1+f_{Pi})dW_{oi} + \sum_{\substack{j=1\\ j\neq i}}^{J} f_{Pj}dW_{oj} + v_P$$

$$\ell = \mathbf{A}\mathbf{x} + \mathbf{v}$$

$$\mathbf{x} = (\delta W_{o1}, \delta W_{o2}, \ldots, \delta W_{oJ})^T$$

$$\ell = (l_{P_1}, l_{P_2}, \ldots, l_{P_N})^T,$$

$$l_{P_k} = \gamma h_{P_k} - C_{P_k i} + \Delta W_o - T_{P_k}^{GGM} - T_{P_k i}^{res}$$

$$\mathbf{A} = \begin{bmatrix} 1+f_{P_1 1} & f_{P_1 2} & \cdots & f_{P_1 J} \\ 1+f_{P_2 1} & f_{P_2 2} & \cdots & f_{P_2 J} \\ \vdots & \vdots & \vdots & \vdots \\ 1+f_{P_N 1} & f_{P_N 2} & \cdots & f_{P_N J} \end{bmatrix}$$

◆ Stochastic model

$$\mathbf{C}_l = g^2\mathbf{C}_h + \mathbf{C}_C + \mathbf{C}_T$$

◆ Solution

$$\hat{\mathbf{x}} = (\mathbf{A}\mathbf{C}_l^{-1}\mathbf{A})^{-1}\mathbf{A}\mathbf{C}_l^{-1}\ell, \qquad \mathbf{C}_x = (\mathbf{A}\mathbf{C}_l^{-1}\mathbf{A})^{-1}$$

NA Simulation Study

◆ Working with residual gravity anomalies

- Remove-restore method

$$T_{Pi}^{grav} = T_P^{GGM} + T_{Pi}^{res} = T_P^{GGM} + \mathbf{S}_P^{res}\Delta g_i^{res}, \quad \Delta g_i^{res} = \Delta g_i - \Delta g^{GGM}$$

$$\mathbf{S}_P^{res} = \mathbf{S}_P^{res} - \mathbf{S}_{P,N_{\max}}^{res}, \quad N_{\max} = \text{max degree of GGM used}$$

- Then the residual Stokes kernel should also be used in the computation of the f_{Pi} coefficients

$$f_{Pi}^{res} = \mathbf{S}_P^{res}\left(\frac{2}{R}\right)$$

◆ Questions to be investigated, for cm-level datum unification:

If we use satellite only GGMs of $N_{max} \geq 180$,

- Will the omission error be small enough to ignore?
- Will the indirect effect be small enough to omit?

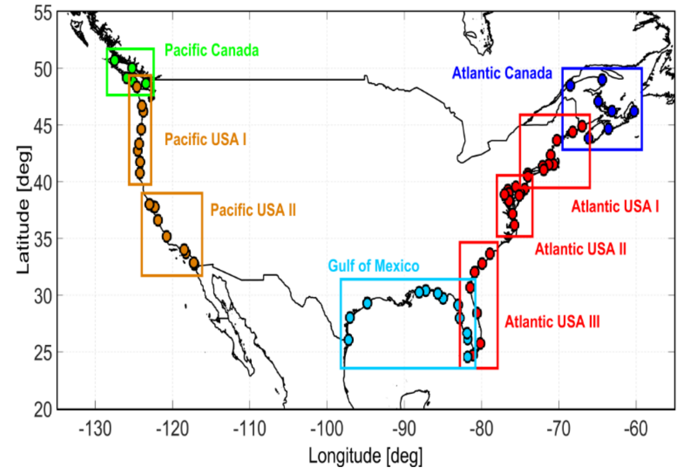
Omission Error (1/2)

◆ Omission error approximated by EGM2008 for $181 \le n \le 2190$

- Tested at NA tide gauges

Region (# of TGs)	Mean (m^2s^{-2})	σ (m^2s^{-2})	Min (m^2s^{-2})	Max (m^2s^{-2})
Canada Atlantic (7)	0.10	3.23	–5.49	3.72
US Atlantic (28)	–0.29	3.63	–8.23	7.06
Canada Pacific (5)	0.59	3.92	–3.53	6.27
US Pacific (17)	0.69	3.23	–4.80	7.25
Gulf of Mexico (13)	0.29	2.55	–6.08	2.74

- Averaging over many points reduces the omission error
- Omission error can reach several dm at individual stations and therefore it should not be omitted

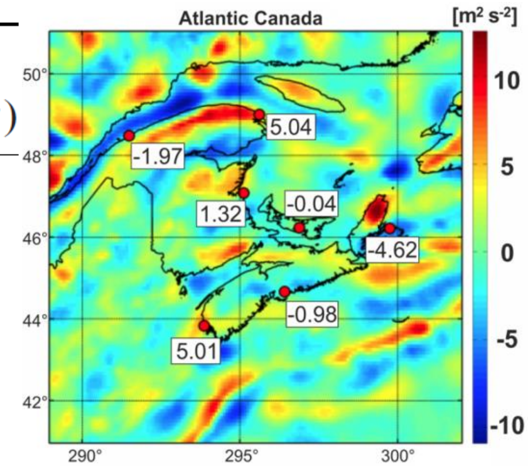


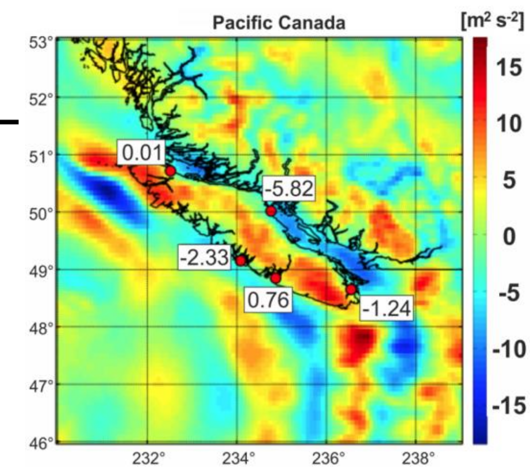
Omission Error (2/2)

◆ Omission error from DIR5 model & 2' gravity/topography grid

- DIR5 N_{max} used: 210 in CA, US; 250 AL; 280 ME

Region	Mean (m^2s^{-2})	σ (m^2s^{-2})	Min (m^2s^{-2})	Max (m^2s^{-2})
Atlantic Canada	0.75	3.00	–3.53	4.52
Pacific Canada	–2.21	2.56	–5.82	0.24
Atlantic USA I	1.05	1.29	–1.38	2.81
Atlantic USA II	–1.60	2.54	–5.59	1.56
Atlantic USA III	0.25	2.89	–3.91	4.62
Pacific USA I	–1.66	2.96	–7.12	3.11
Pacific USA II	–1.12	1.80	3.49	2.32
Gulf of Mexico	–0.23	1.59	–3.70	2.86





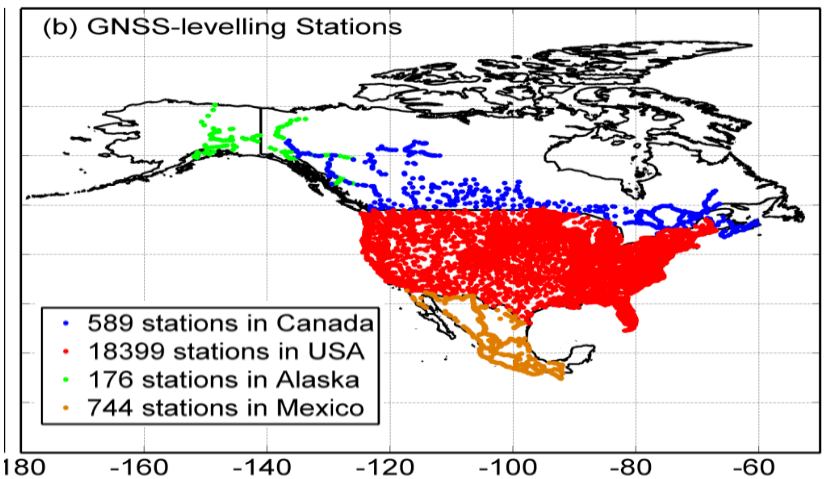
- Same conclusions as previous test

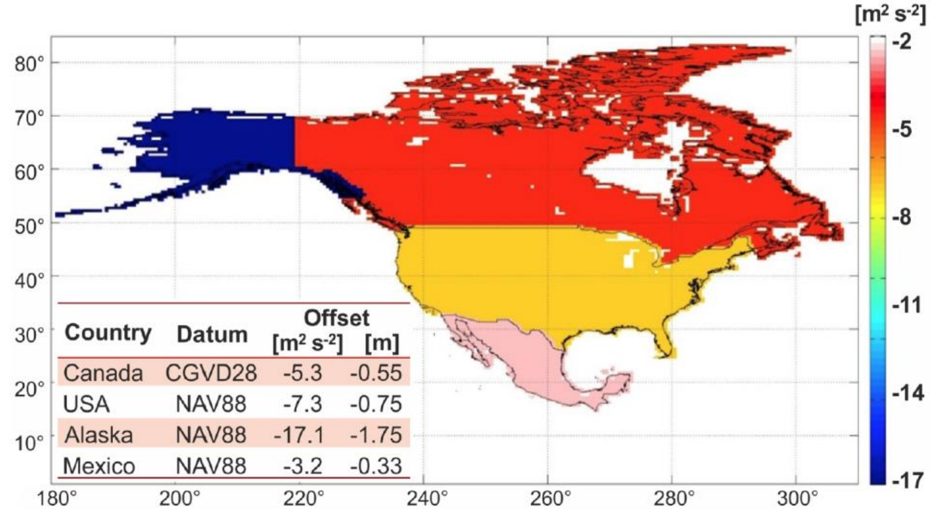
◆ Conclusions have also been verified at GNSS / levelling stations in Canada, USA, Mexico

Indirect Bias Term (1/2)

◆ Evaluation of Indirect Effect

- ❖ Used mean offsets on a 30’ grid w.r.t. the W_o = 62 636 853.4 m²/s² potential
- ❖ Computed at GNSS/levelling stations for various N_{max} values in DIR5 and $\mathbf{S}_p^{res}$



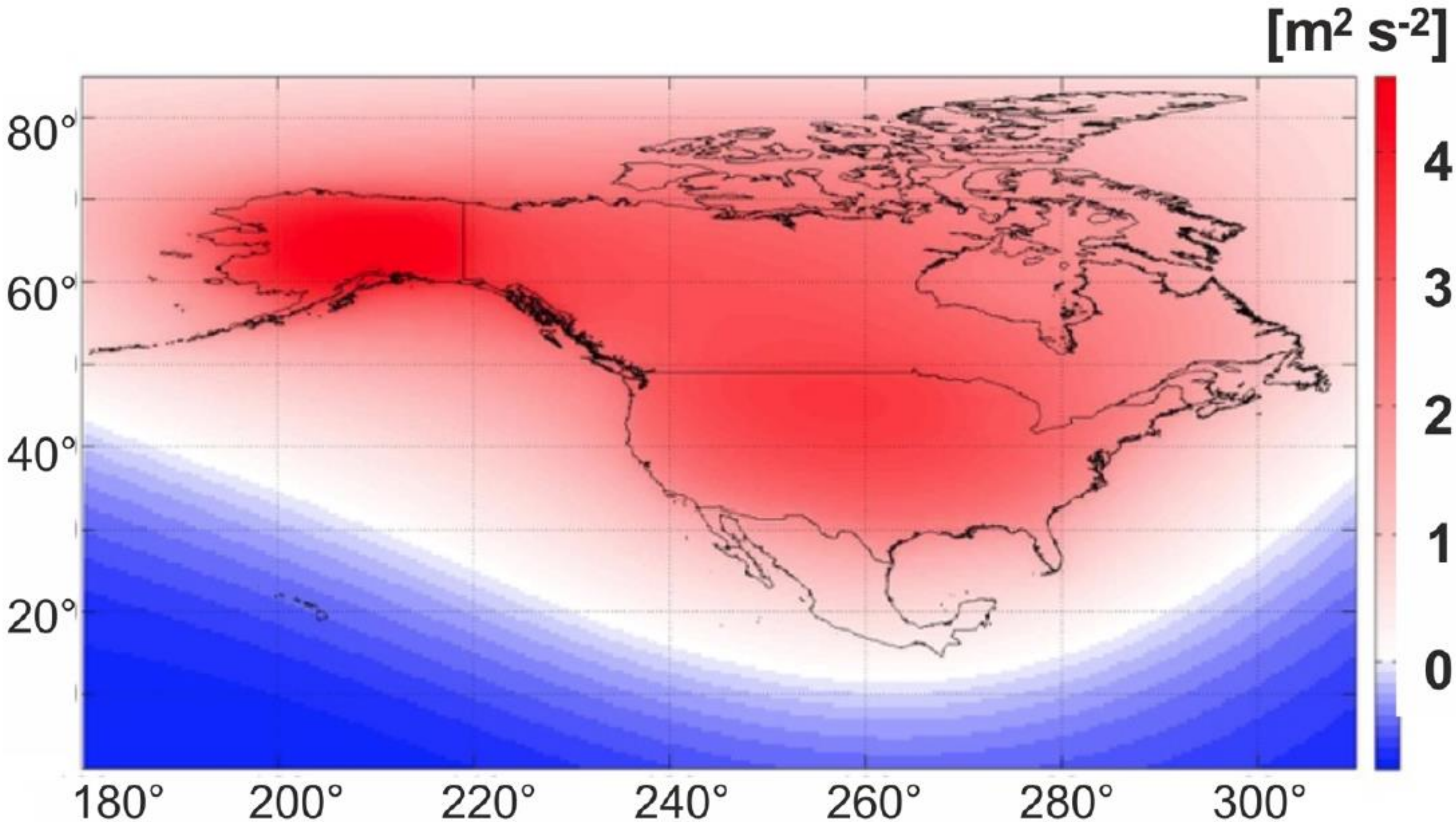


Country	Datum	Offset [m² s⁻²]	Offset [m]
Canada	CGVD28	-5.3	-0.55
USA	NAV88	-7.3	-0.75
Alaska	NAV88	-17.1	-1.75
Mexico	NAV88	-3.2	-0.33

◆ Results (on next page) show that:

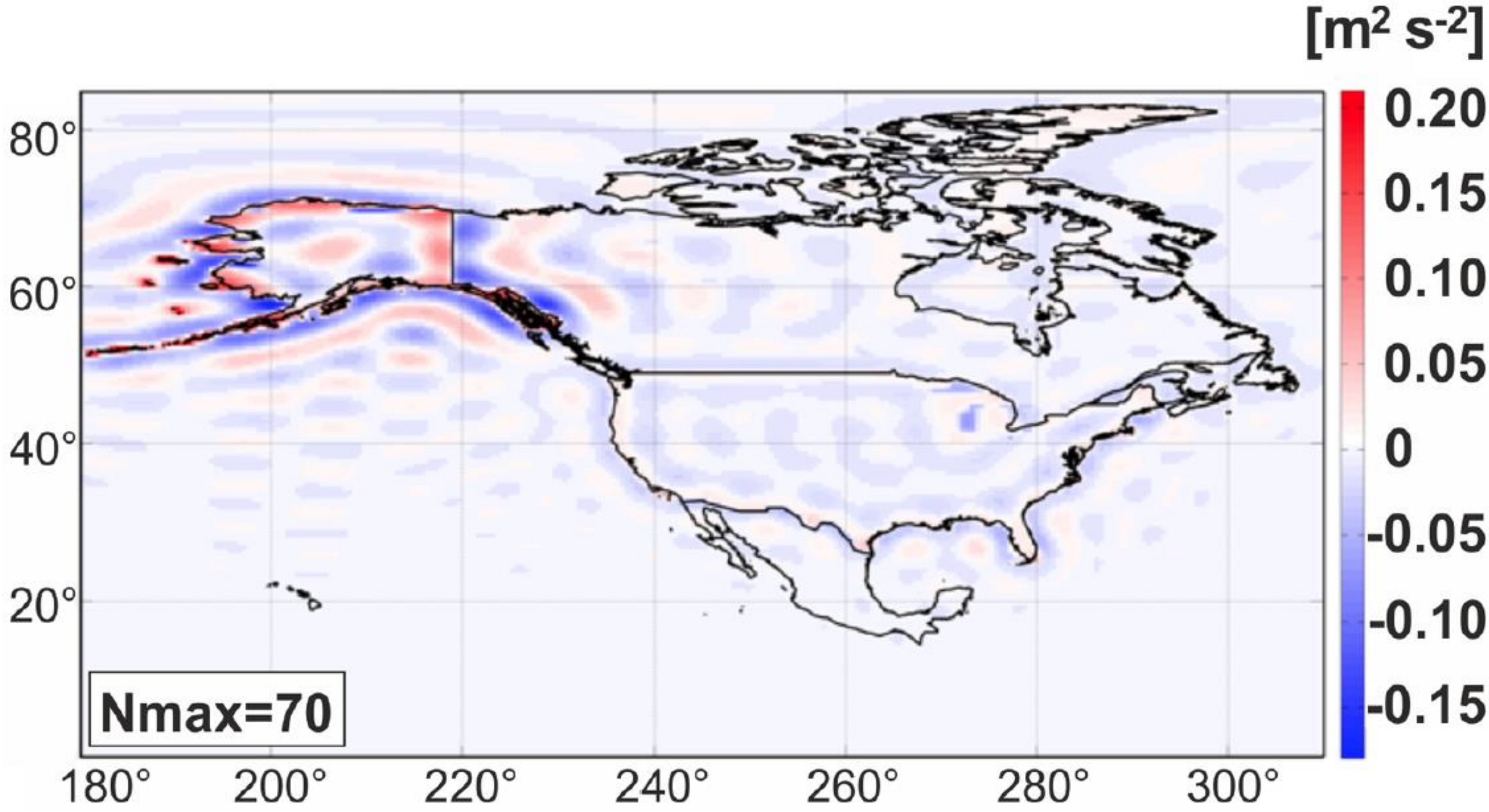
- ❖ < 1 cm when the satellite-only GGMs are used to d/o ≥ 180 – can be omitted
- ❖ Observation equations are greatly simplified (can set all f_{Pi} = 0)
  - ◆ Datum offsets are then just the weighted averages of all station δW_{oi} values

Indirect Bias Term (2/2)

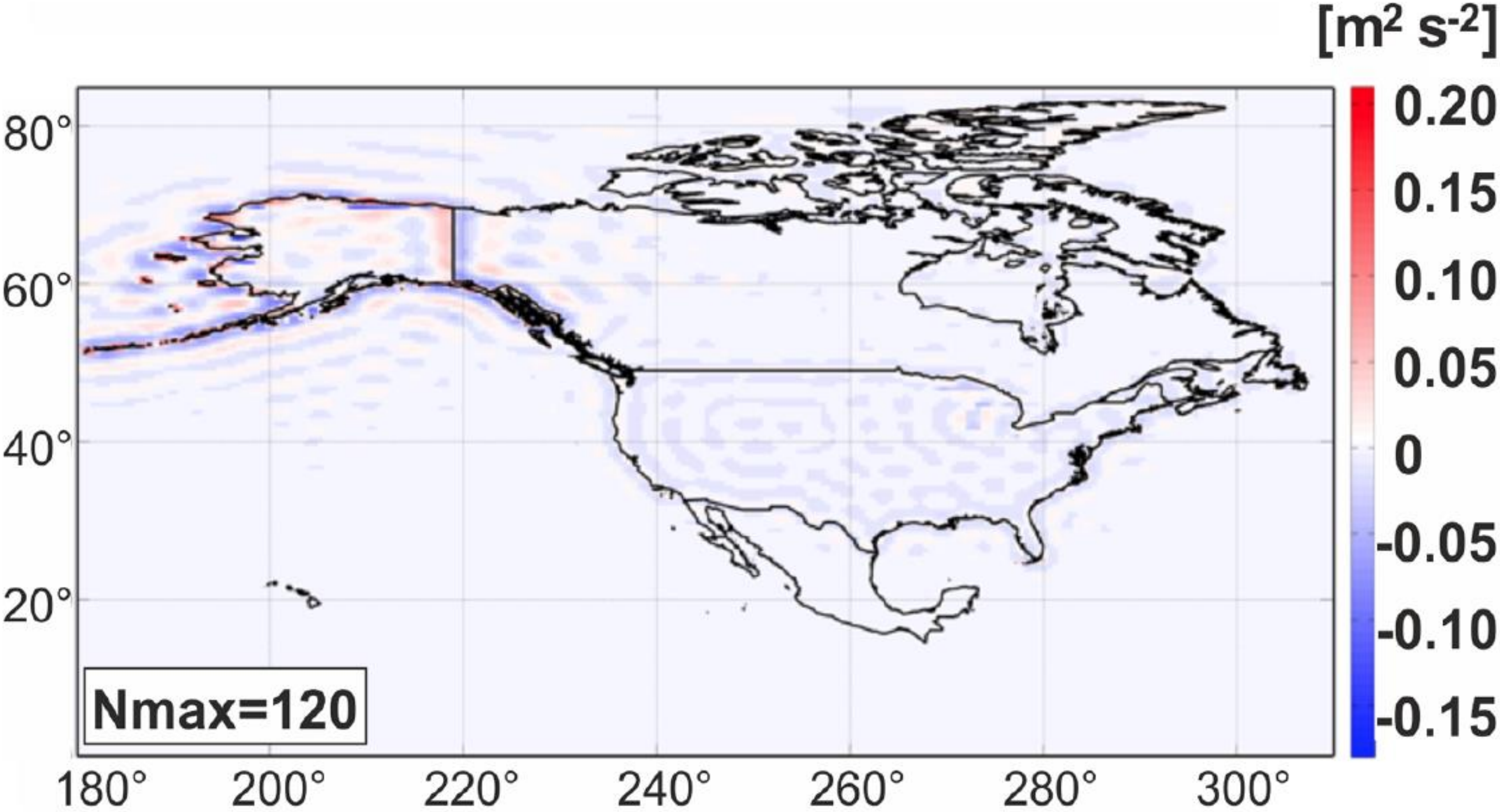


❖ Indirect bias term computed with the original Stokes kernel > 40 cm !

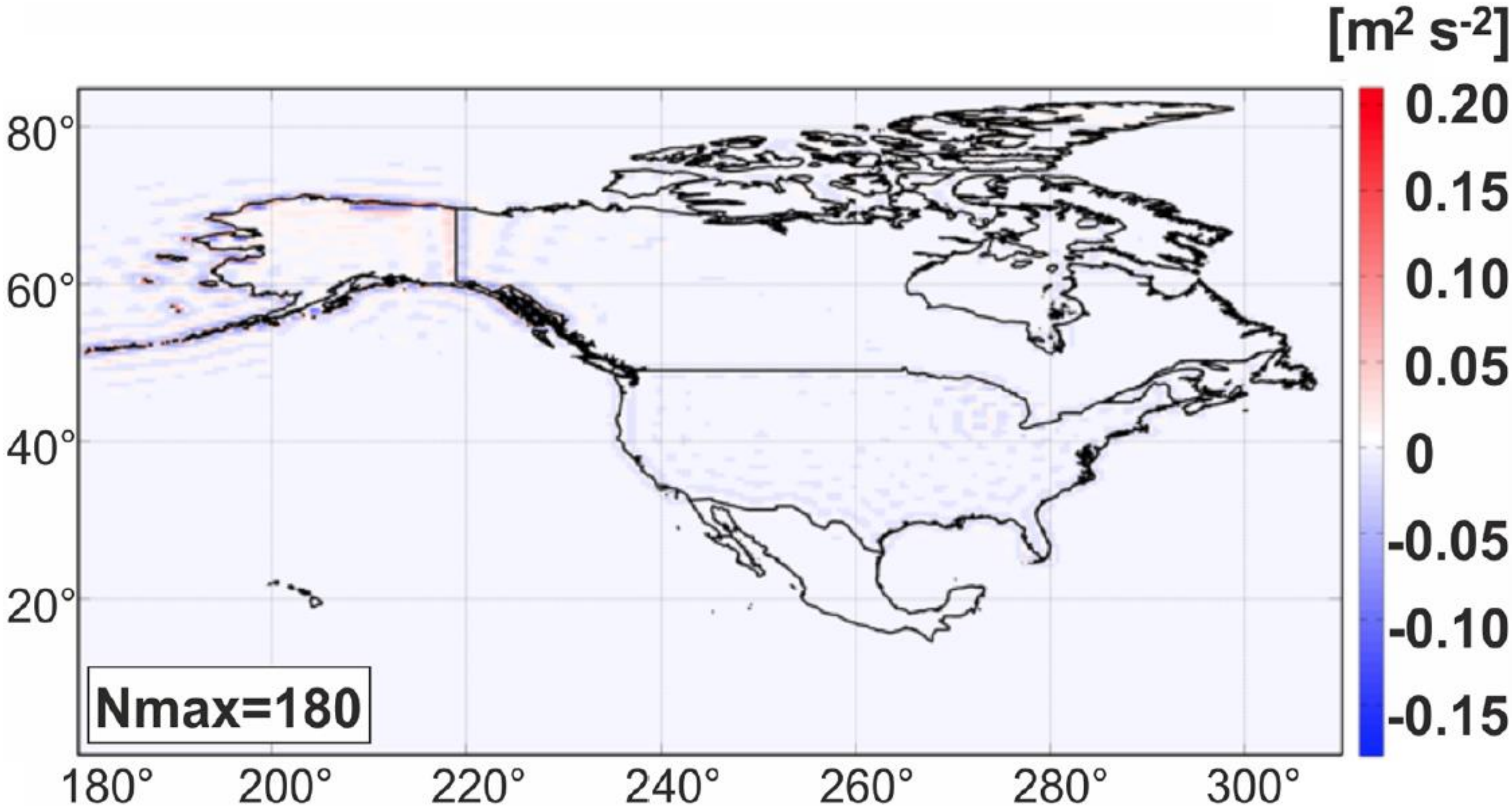
Indirect Bias Term (2/2)



Indirect Bias Term (2/2)



Indirect Bias Term (2/2)



❖ It is insignificant (max value < 1 cm) when the satellite-only GGMs are used to d/o ≥ 180

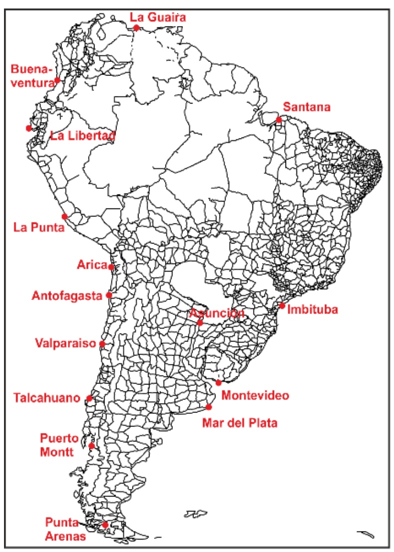
SA Vertical Datum Unification

◆ Required data for the formulation of the observation equations

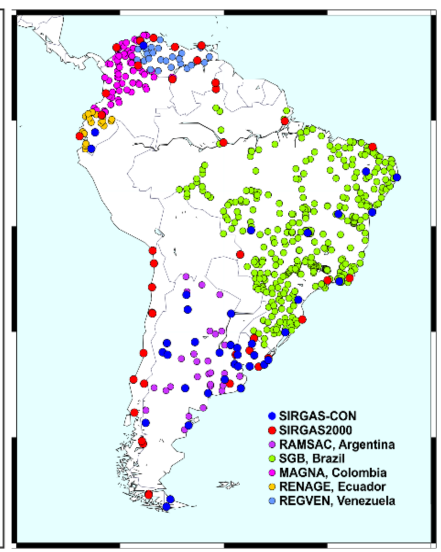
- Terrestrial gravity anomalies distributed homogeneously and in a high geographical density for the estimation of the disturbing potential T.
- Geopotential numbers C_P derived from levelling with gravity corrections in land areas.
  - Optional: geopotential numbers from geostrophic or steric levelling or satellite or altimetry-based mean dynamic topography (MDT) models in ocean areas.
- Ellipsoidal heights h derived from GNSS positioning in land areas and from satellite altimetry in ocean areas.
- Border levelling points with geopotential numbers referring to neighbouring vertical datums $C_{P,i}$, $C_{p,i+1}$.

Observables available in South America

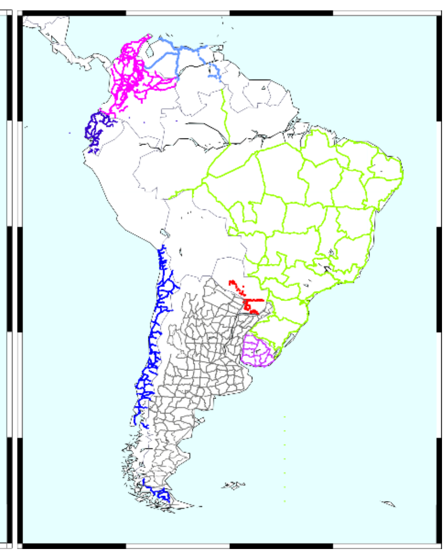
- 663 geometric reference stations connected with the national vertical datums.
- 7 international connections between neighbouring national vertical datums.
- 14 reference tide gauges with MDT values.
- All these stations with: known geopotential numbers, ellipsoidal heights and anomalous potential values (all of them with uncertainty values).



Reference tide gauges



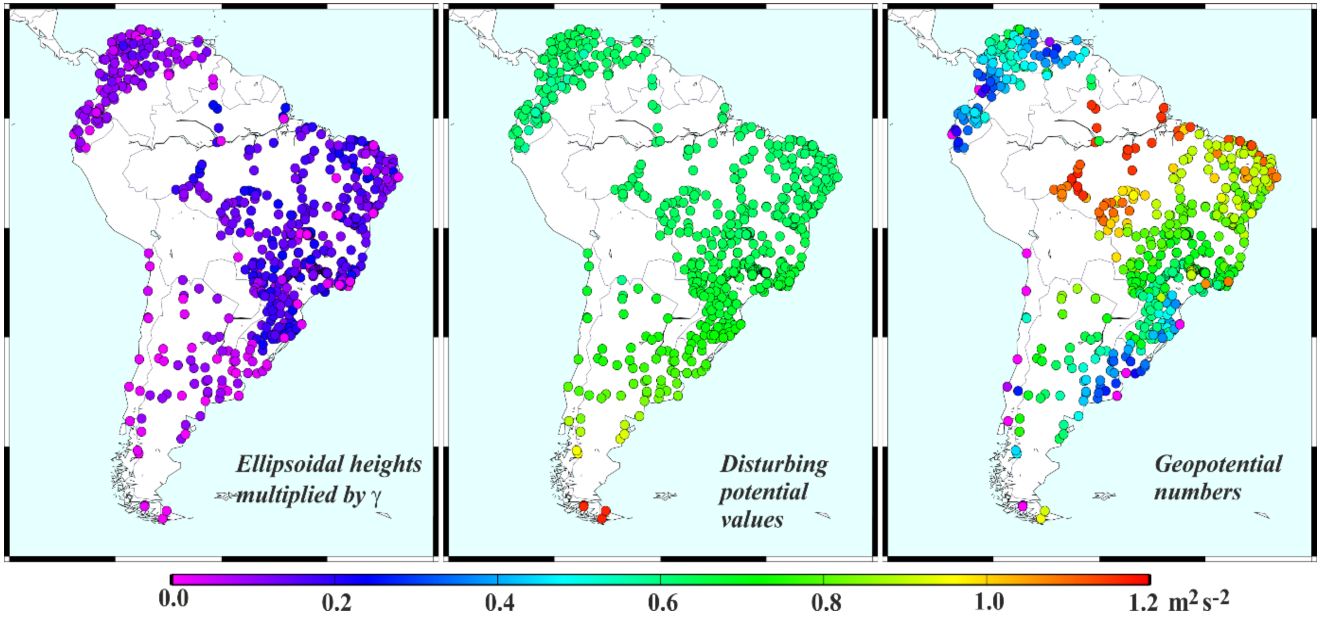
Geometric stations



Levelling networks

◆ Empirical procedure

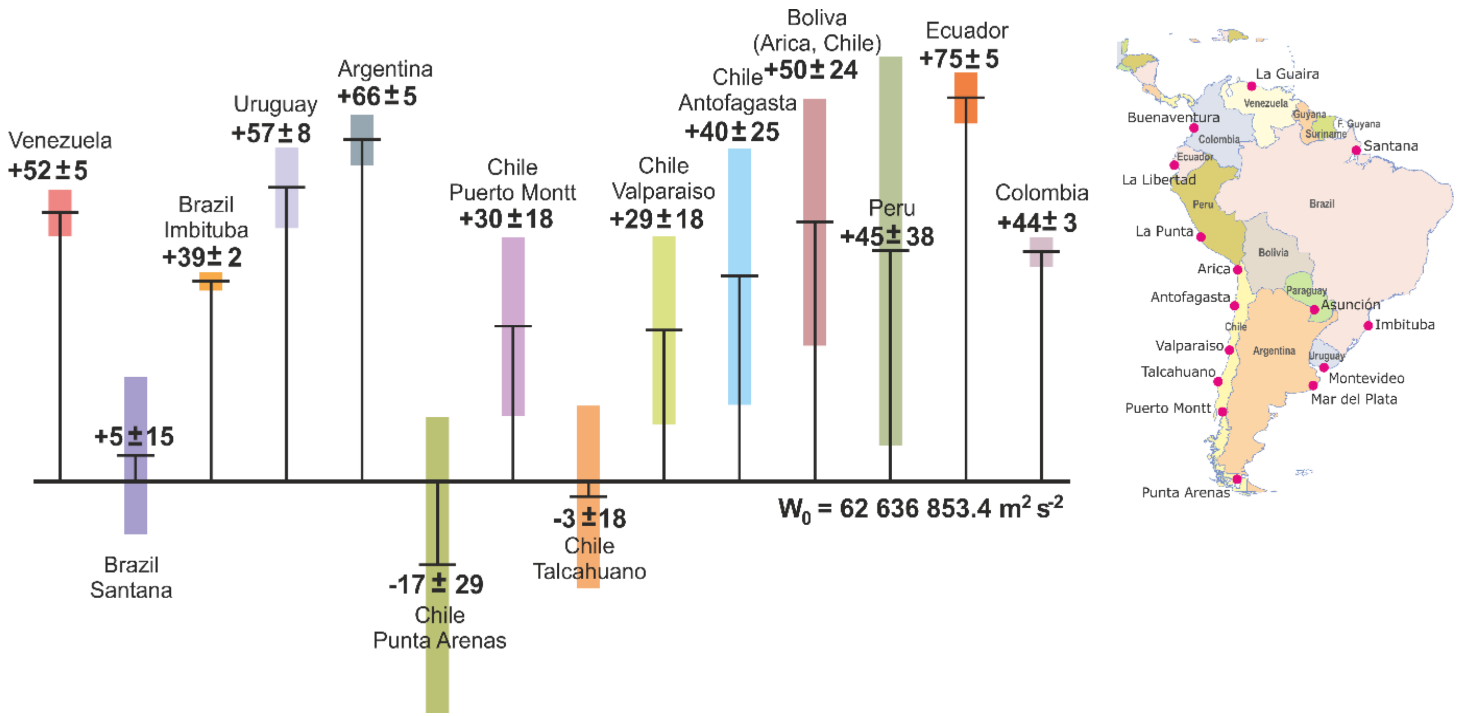
- ❖ Harmonisation of the input data, e.g.; all heights in zero tide system and at the same reference epoch, the same reference GGM (n=200) for the estimation of *T*.
- ❖ Weighted least-squares adjustment using the inverse of the variances of the input data and rigorous error propagation analysis.



Standard deviation of the input data used for the vertical datum unification in South America.

Results

Vertical datum parameters in [cm] with respect to W_0 = 62 636 853.4 m²s⁻².



- Accuracy:
  - ±0.5 m^2s^{-2} (±5 cm) in regions with a high number of observations (Argentina, Brazil, Colombia, Ecuador, Uruguay and Venezuela).
  - ±2 to ±4 m^2s^{-2} (±20 to ±40 cm) in regions with a small number of observations (Bolivia, Peru and the southern part of Chile).

Conclusions (1/2)

- The omission error should always be accounted for, in particular in regions with poor distribution of TG or GNSS-levelling stations
- The indirect bias term can be omitted for a GGM of DO 180 in North America (below 1 cm)
  - Then the LVD offset is a (weighted) mean of the discrepancies between the geometrically-derived and gravimetric geoid heights
- As the estimation of the datum parameters should be as reliable as possible, only geodetic stations of highest quality should be considered for the vertical datum unification.
- Possible sources of inconsistency should be removed; i.e., standardised geodetic data is required; for example, geometric coordinates should refer to the same ITRF and be given in the same tide system and reference epoch as the geopotential numbers and gravity field model.

Conclusions (2/2)

- Since the disturbing potential should be estimated with high-precision, it is recommended to compute (a) the long wavelength component ($n \leq 200$) using a common GGM, and (b) the short wavelength component ($n > 200$) by the combination of terrestrial gravity data and detailed terrain models. The use of a GGM is not sufficient.

- After a standardisation of the input data used in the unification of the South American height systems and a rigorous error propagation analysis, we demonstrate that the vertical datum parameters can be estimated with accuracy better than ±5 cm in well-surveyed regions and some decimetres (± 40 cm) in sparsely surveyed regions

- Once a first estimation of the vertical datum parameters is available, the height-related observables (geopotential numbers, terrestrial gravity anomalies) should be re-computed and used to iterate the GBVP solution. This procedure should be repeated until sub-mm differences are obtained between consecutive iterations.